Development of a 3D Semi-Analytical Method for Inverse Problems in Elastic Wave Propagation in Anisotropic Media

The modelling of elastic wave propagation in anisotropic media is of major interest for many scientific and engineering applications, including non-destructive testing, medical ultrasound in anisotropic tissues, and acousto-optic modulation in anisotropic crystals, where ultrasonic waves are used to control light diffraction and beam steering. With traditional numerical approaches, such as the Finite Element Method, the resolution process requires substantial memory resources and is highly time-consuming, especially for anisotropic materials.

This research aims to develop a new semi-analytical method based on angular spectrum [1-3] to solve **inverse** problems in elastic wave propagation, in elastic anisotropic materials then in piezoelectric anisotropic materials. The objective is to determine the source generation conditions required to obtain a target acoustic sound field in anisotropic crystals. Considering that the waves are generated by a planar source located on the surface of a solid, the problem can be then formulated as a back-propagation problem. The angular spectrum method would be a very efficient solution way to solve the wave back propagation in anisotropic materials.

Based on this principle, the research tasks consist in adapting of the plane-wave angular spectrum method to anisotropic solids, developing of computational software enabling the definition of the source geometry necessary to generate the target acoustic field distribution and to visualize and analyse the three-dimensional elastic fields. This framework is particularly relevant for wave-field design in anisotropic media and for the optimization of devices relying on controlled ultrasonic propagation.

The proposed developments will be compared with experimental results that will be obtained in anisotropic materials such as paratellurite.

Based upon the research topic proposed here, the investigations will be organised in various tasks:

1. Adapt plane-wave angular method to 3D anisotropic elastic materials
2. Adapt plane-wave angular method to 3D anisotropic piezoelectric materials
3. Develop a back-propagation algorithm
4. Develop an iterative method for the design of the generation conditions
5. Develop an optimisation procedure to generate an electrode design

To validate the theory and test the software, experimental investigations will be conducted on acousto-optic filters made of highly anisotropic crystalline material such a paratellurite, among others.

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