Development of a 3D semi-analytic method for the elastic wave propagation in anisotropic media

Modelling the propagation of elastic waves in anisotropic materials has many applications. Usual tools utilised to study the elastic wave generation and propagation are based on traditional numeric approach such as the Finite Element Method. However, though these methods are well-known, the resolution process requires heavy memory consumption and is highly computational-time consuming for anisotropic materials. Here, a new elegant semi-analytical method based on angular spectrum will be studied and developed.

Mathematically, modelling the elastic wave for a particular problem involves finding solutions satisfying both the boundary conditions and the elasto-dynamics equation. For isotropic media, the general solution of the hyperbolic equation can be obtained in integral form via the Lamé expression of scalar and vector potentials [1] in displacement field. However, the Lamé solution is not applicable to anisotropic media. Neither general nor simple solution can be established. In many physical applications however, such as planar ultrasonic sensors, surface wave devices, layered media, interface reflection/refraction, etc., the waves are generated by a planar source located on a solid surface. In these situations, the question is reduced to a half-space problem [2] and the angular spectrum of unbounded plane waves would be a very efficient way to solve the wave propagation in anisotropic materials. The solution can be considered as the superposition of three orthogonal plane waves (one q-P-wave and two q-S-waves) propagating in all the direction (3 dimensions) while the boundary conditions are satisfied by adjusting the complex amplitude of these waves.

Based upon the principle above proposed, this research works consist of

1. The determination of complete plane wave angular spectrum in half space by solving the eigen-modes of Christoffel equation, including both propagating (real wavenumber) and evanescent (complex wavenumber) waves. Some particularities relative to material anisotropy, i.e. cuspidal singularity, negative group velocity should be treated.
2. Coupling different wave modes of the angular spectrum by applying the boundary conditions to obtain the diffraction wave beam, and taking into account the eventual singularities due to the generation of the surface waves which could lead to numerical instability.
3. Realization of a computation software (preference in python language or matlab) permitting the definition of the material characteristics (parameters) with interactive interfaces, slowness surfaces and dispersion curves plotting, and 3D elastic field visualization and analysis.

To valid the theory and test the software, experimental works will be conducted, in the case of acousto-optic modulator of highly anisotropic crystalline material, among others.

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