Title

Observation analysis and observer design for switched system on non-uniform time domain

Keywords: Switched systems, Time scale theory, Observer design, Observation, Fault detection

Background, context:

The time scale theory was firstly introduced by Stephan Hilger in his Phd thesis [9] in 1988 in order to unify the theory of continuous dynamical systems and discrete dynamical systems. If $\mathbb{T} = \mathbb{R}$, dynamical equations reduce to standard continuous differential equations. When $\mathbb{T} = h\mathbb{Z}$, (h is a real), they are reduced to classical difference equations. In addition, between these two extreme cases, there are other interesting time domains that are a mixture between the continuous and discrete time (as a time domain formed by a union of disjoint intervals), or a discrete time domain with a non-uniform step size, such as the time scale $\mathbb{T} = \{t_n\}_{n \in \mathbb{N}}$, called harmonic numbers with $t_n = \sum_{k=1}^{n} \frac{1}{k}; \quad n \in \mathbb{N} =$ Cantor set, etc.

Switched systems are systems involving both continuous and discrete dynamics. They consist of a finite number of subsystems and a discrete rule that dictates switching between these subsystems. They have been widely studied during these two last decades because they can describe a wide range of physical and engineering systems [1]. Stabilization and observer design have been widely studied for this class of systems. They are usually categorized into two separated directions depending on whether each subsystem is continuous-time or discrete-time. The extension of these results to switched systems evolving on a non-uniform time domain is still an open problem (see Fig. 1).

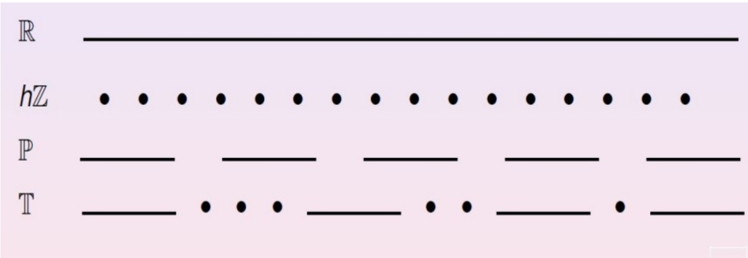

Fig. 1: a. Examples of time domains (the last two ones are non-uniform)..

The time scale theory was found promising because it demonstrates the interplay between the theories of continuous-time and discrete-time systems [2]. It leads to a new understanding and analyzing of dynamical systems on any non-uniform time domains that are closed subsets of R. Time scale dynamic equations reduce to standard continuous differential equations (resp. standard difference equations) when the time scale is the continuous line (resp. homogeneous discrete domain). Besides these two extreme cases, there are many interesting examples considering nonhomogeneous time scales [3-8].

The subject of this PhD focuses on observability analysis and observer synthesis for hybrid linear systems evolving in a non-uniform domain using the time-scale approach. The time-scale approach is a method for analyzing systems that exhibit behaviors on very different time scales. It allows to decompose the systems into different modes of operation, each with a characteristic dynamic, and to analyze them separately.

The objective of the work is to develop a method of observability analysis for hybrid linear systems evolving in a non-uniform domain, using the time-scale approach. This method will allow to determine the modes of operation of the system that are observable, i.e. those for which it is possible to reconstruct the state of the system from measurements.

Then, the work will focus on observer synthesis for hybrid linear systems evolving in a non-uniform domain. The objective of observer synthesis is to design a system that can estimate the state of the system from measurements, even if some modes of operation are not observable. The method developed in this internship will allow the synthesis of observers for hybrid linear systems evolving in a non-uniform domain, using the time-scale approach.

The thesis will involve the implementation of these methods on numerical examples and/or concrete case studies. The candidate will also have the opportunity to deepen his knowledge on hybrid linear systems and the time-scale approach, as well as on modeling and simulation tools.

In addition, fault detection and isolation (FDI) for systems evolving in a non-uniform time domain remains a theoretical area yet to be investigated. Thanks to control theory techniques, the use of observers, which can be considered as software sensors, will allow to reduce the number of physical sensors used on the system and to generate the residuals that will be used as indicators of the presence of faults. Missing information due to missing sensors is reconstructed using data from the remaining sensors and knowledge of the system. The combination of condition monitoring and the use of observers will result in better performing and less expensive systems.

Research subject and work plan:

The purpose of this thesis is the design of new theoretical tools for the observability analysis and observer design for switched systems on non-uniform time domain using the time scale theory with nonlinearity. The objective is to develop algorithm of fault detection for such systems evolving in non-regular time domain.

The main further works will concern:

- To analyze the observability of switched systems on non-uniform time domain using the time scale theory. Indeed, observability for switched systems is defined in various ways, usually via indistinguishability relation. Hence, they mean the same for continuous-time and discrete-time systems. The objective is to show that some of these concepts can be studied in a unified way using the time scale theory.
- To design observers for systems on non-uniform time domain using the time scale theory, some conditions will be derived to guarantee the exponential stability of the estimation error on time scales with bounded graininess function when all the subsystems are exponentially stable. The design of common Lyapunov function or multiple Lyapunov function will be useful to derive appropriate observers.
- To develop algorithm of Fault Detection and Isolation of switched systems in time scale. The used approach will be based on the use developed observers.

Requested knowledge

- Basic knowledge of linear systems and control theory;
- Good knowledge in applied mathematics;
- Basic knowledge in numerical analysis and simulation;
- Programming skills (MATLAB, Simulink);
- Ability to work independently and communicate effectively, especially in English.

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