Ph.D. Project:

Robust multi-objective and multi-period optimization of virtual network embedding

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Overview of the project

Given a graph representing a physical network with limited node and arc resource capacities and a set of virtual networks demanding physical node and arc resources, the Virtual Network Embedding Problem (VNEP) is a graph optimization problem that requires to decide how to embed the virtual networks in a physical network, by mapping the virtual nodes and arcs to physical nodes and paths, satisfying the resource demand while respecting the capacity of network elements (see an example of graph embedding in Fig. 1). The VNEP is a well-known NP-hard problem associated with important applications arising in various contexts, such as communication and transportation networks. The objective of this Ph.D. Thesis project is to pursue a unified view on optimal virtual network embedding by investigating the development of innovative optimization models that take into account the multi-faceted nature of the VNEP, namely jointly modeling its a) multi-period nature (virtual network requests evolve over time); 2) data uncertain nature (the resource consumption of requests is not known in advance); 3) multi-objective nature (the embedding must typically pursue contrasting objectives like minimization of resource consumption and maximization of embedded requests). For solving the resulting complex optimization problem, new advanced robust optimization *matheuristics* combining exact and heuristic solution approaches will be developed and tested computationally.

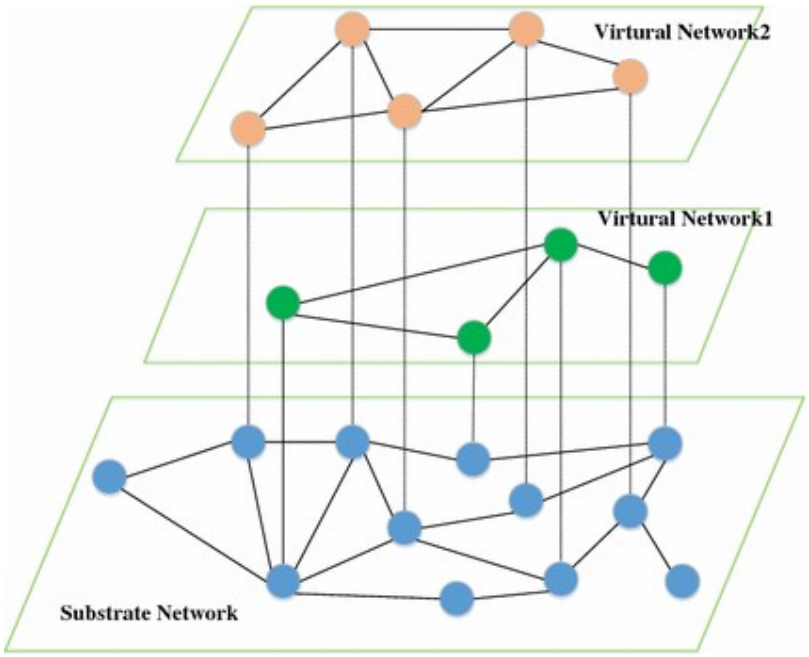

Figure 1: an example of virtual network embedding, with nodes and arcs of two virtual networks mapped to nodes and paths in the substrate network [ZhTh17].

Over the years, optimal virtual network embedding has received a lot of attention and has been extensively studied both from a theoretical and an applied point of view (see e.g. the survey [FiEtAl13] for an effective introduction and review of major works). However, the majority of works has focused on considering just single optimal decision aspects arising in the VNEP. For example, most of the works consider a single-period problem that neglects the dynamic nature of the embedding, which takes place over a time horizon in which new virtual network requests appear while old requests have achieved their scope and are cancelled. Other works only consider a single objective to optimize, neglecting the fact that the VNEP is intrinsically multi-objective and requires to combine multiple contrasting objectives (for example, minimizing resource usage and maximizing request acceptance). Finally, most of the works do not consider the uncertain nature of the VNEP, which contains multiple sources of data uncertainty (e.g., not knowing exactly the features of future virtual networks and neglecting the possibility of network element failures), and focus on deterministic versions of the problem.

Our objective is to fill the gap highlighted above by investigating the development of an optimization approach that *jointly* takes into account all *the data uncertain, multi-period and multi-objective decision aspects* of the VNEP and proposes new innovative optimization models and algorithms for representing and solving it.

Expected Main Phases of the Ph.D. Project

The Ph.D. project will be articulated in the following 4 main phases.

Phase 1 – Literature study and identification of the virtual network embedding problem. After an exhaustive review of the state-of-the-art concerning the VNEP, this phase will be aimed to reach a clear description of the robust multi-objective multi-period optimal virtual network embedding problem that will be addressed. A critical task will be also to identify how to take into account uncertain aspects of virtual network requests generated by users (e.g., amount of resource requested), which critically affects the design of the embedding over the complete planning period.

Phase 2 - Modelling the embedding problem and the presence of data uncertainty. The first objective of this phase will be to define a reference *deterministic* version of the multi-objective multi-period problem, namely a version of the problem not considering data uncertainty. We expect to derive a reference Mixed Integer Linear Programming optimization problem, to represent binary embedding decisions and continuous resource assignment and commodity flow decisions. The second objective of the phase will be to derive a *robust counterpart* of the problem, namely a modified version of the deterministic problem that takes into account the presence of data uncertainty. The methodology that we expect to adopt to tackle data uncertainty is Robust Optimization (RO) [BaEtAl14, BeBrCa11]), which takes into account data uncertainty under the form of hard constraints that cut off all solutions not protected against deviations in the input data of the problem.

Phase 3 - Developing optimization algorithms for the solution of the problem. As indicated by many studies (see [FiEtAl13]), even solving a single-objective and single-period version of the virtual network embedding problem may prove very challenging even for state-of-the-art optimization solvers, due to the rapid explosion in size associated with mapping virtual arcs to physical paths. As a consequence, we realistically expect that it will not be possible to solve the developed models by a straightforward application of an optimization software and the definition of ad-hoc exact and heuristic solution algorithms will represent a crucial part of the Ph.D. investigations. In particular, it is intended to develop *matheuristics*, i.e. hybrid solution algorithms that attempt at getting the best of both the exact and heuristic world, integrating exact algorithms, which guarantee convergence to an optimal solution but typically perform slow, with heuristic algorithms, which are fast and efficient but do not provide guarantees about the quality of produced solutions.

Phase 4 - Definition of realistic problem instances and computational tests. As final step, it is intended to identify relevant application use case of virtual network embedding, arising in real-world contexts, and provide a realistic assessment of the performance of the new proposed approaches.

Candidate's profile:

Applicants must have a Master Degree (or equivalent) in Computer Science, Applied Mathematics, Industrial Engineering or any related discipline. Applicants should demonstrate proficiency in English, good programming skills and knowledge of mathematical optimization. Experience on the development of (optimization) models and algorithms for communication networks will be appreciated.

Documents required to apply:

Please send to hicham.lakhlef@hds.utc.fr and d.andreagiovanni@hds.utc.fr

- a curriculum vitae
- a motivation letter
- at least two references and/or recommendation letters
- a statement of research interests and experience (if any)

Essential References:

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[ZhTh17] M. Zhang, X. Tang, "Hop-Limit Path Mapping Algorithm for Virtual Network Embedding", Wireless Personal Communications 85, 2033-2048 (2017)