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Description of the research work proposed for a PhD

Title: Non-asymptotic Estimation Approaches for Fractional Order Nonlinear Systems

Keywords: Nonlinear Fractional Order Systems; Nonlinear Observer Normal Forms, Non-asymptotic Estimation Approaches.

Subject:

I. Context

I-1. Fractional order time delay systems

Fractional calculus was introduced in many fields of science and engineering long time ago. It was first developed by mathematicians in the middle of the ninetieth century. During the past decades, fractional calculus has gained great interest in several applications [1]. For instance, fractional order systems and controllers have been applied to improve performance and robustness properties in control design.

I-2. Observer design

For engineers, there is always a need of knowing the states of a system in order to make important decisions, to control the system, or to predict reliably its future states. A simple and sure method for knowing the states is to measure them directly. However, in general, it is not feasible or even impossible to directly measure all of the states of a system. An alternative method for knowing the states of a system instead of measuring all of them is to develop a model for the system. If the model is perfect, the states can be accurately estimated when the inputs and the initial conditions of the system are known with good accuracy. However, the initial conditions are never known and some inputs are unknown (external disturbances for example). Hence, the estimation by a model is effective only when the model error is quite small, the effects of initial conditions are not significant and the inputs are known. In order to solve these problems, the concept of observers (called also software sensors) has been introduced to efficiently estimate the states of a system [2].

Observer design for dynamical systems has received many attentions since the pioneer works of Luenberger and Kalman [3]. It concerns the reconstruction of the states of a system based on the available measurements and the model of the system (which can be uncertain). In order to design an observer, the concept of observability is important, which concerns the ability of estimating the states using the available data of inputs and outputs of a system.

I-3. Nonlinear observer normal forms

The observer design is useful to estimate the states of a system and is important in control theory. However, it is usually difficult to design an observer for a nonlinear system. In order to solve this problem, the so-called Nonlinear Observer Normal Forms (NONFs) have been introduced for nonlinear systems during the 1980s [4]. The idea is to apply geometrical method to transform a nonlinear system into a NONF on which existing observer methods can be applied. Recently, the theory on the NONFs has been widely developed and promising results have been obtained in several areas, such as automotive, robotics, electronics, electrical engineering, and bio-medical [5,6,7].

I-4. Non-asymptotic estimation approaches

Recently, a type of fast estimation methods for simultaneous parameters and state estimation for continuous integer order linear dynamical systems have been proposed [8,9]. These methods are algebraic, thus non-asymptotic (in other words, they allow a finite-time convergence), and robust against corrupting noises without knowing their statistical properties. Thanks to these advantages, they have been applied in many fields, such as signal processing, control, robotic, etc. Very recently, these methods have been applied for fractional order linear systems [10,11,12,13].

II. Objective and expected results

The main objective of this thesis is to non-asymptotically and robustly estimate the state of fractional order non-linear systems in noisy environment. For this purpose, the considered systems will be transformed into NONFs, based on which the non-asymptotic estimation approaches can be applied. The plan of the proposed thesis is outlined as follows:

- Study existing references on the NONFs and give improved the results by considering different NONFs;
- Extend the results on the NONFs from integer order systems to fractional order ones;
- Study existing references on the non-asymptotic estimation approaches for fractional order linear systems and obtain improved results;
- Extend the obtained results on the non-asymptotic estimation approaches from fractional order linear systems to non-linear ones.

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