THESIS TITTLE: Non-polynomial functions and polynomial invariants for modelling anisotropic hyperelastic materials

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SUBJECT

This subject concerns continuum mechanics, computational mechanics, engineering science and health. It is very promising for the coming years given the growing needs in understanding how the human body works (pharmaceuticals, therapeutic, medical prosthesis, ergonomics, diseases related to the workplace).

These thirty past years, many models have been developed to represent the behaviour of soft biological tissue. The thesis of Anh Tuan Ta [22], conducted under the supervision of F. Peyraut, allowed for example to exhibit a new class of anisotropic invariants thanks to the mathematical theory of polynomial invariants [20, 21].

By using this new class of invariants, Renye Cai has succeeded in her thesis [28], also conducted under the guidance of F. Peyraut, to build new anisotropic hyperelastic strain energy densities modelling materials with a one or a four fibers-family [23, 26, 27].

These new models are numerically efficient but include numerous material parameters whose identification requires of lot of costly experimentations. Moreover, it is difficult to give a physical interpretation to all of these material parameters. Find simpler models with less material parameters is therefore a challenge. To address this issue, we propose to replace the polynomial expansion currently used in our models by non-polynomial functions (which must be determined) involving the polynomial invariants with a low number of material parameters, and providing at least the same numerical accuracy as our previous models.

A cooperation with Professor Zhi-Qiang Feng (former Head of the Laboratory of Mechanics at the University of Evry – Val d'Essonne in France) is expected. A close cooperation exists for many years between Professors Feng and Peyraut for the numerical study of hyperelastic laws. In this context, many hyperelastic laws have been investigated and some of them implemented in the finite element software FER [1-19, 23-27]. Figure 1 shows some examples of hyperelastic computations performed with FER, with four standard models. The three first (Blatz-Ko, Ogden, and 3^{e}) are isotropic while the fourth (HGO model) is anisotropic.

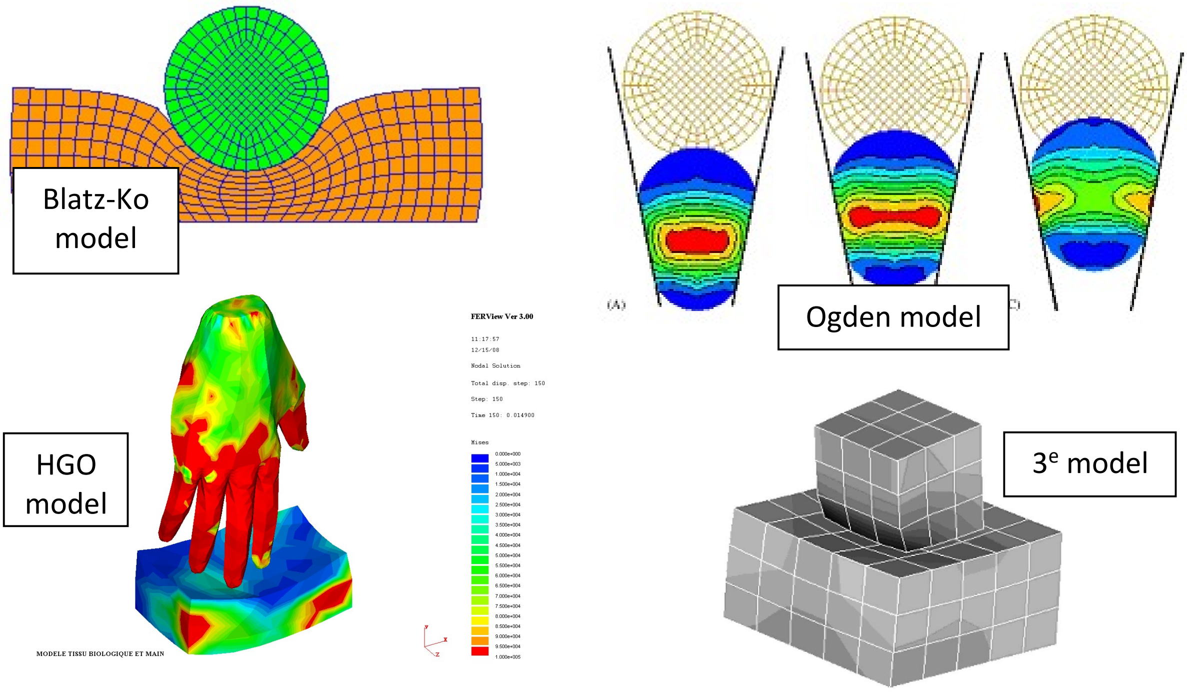

Figure 1 – *Hyperelastic computations performed with the finite element software FER*

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