Ph D proposition  
CSC (China Scholarship Council)  

Title: Multicriteria tool to enhance thermal building renovation  

Topic CSC : VI-4 Intelligent construction  

Supervisor:  
Stéphane Ginestet, Assistant Professor  
E-mail address: stephane.ginestet@insa-toulouse.fr  
Phone: +33 5 61 55 99 14  
Fax: +33 5 61 55 99 00  

Laboratory: LMDC Laboratoire Matériaux et Durabilité des Constructions  
Institutions : INSA Toulouse, Institut National des Sciences Appliquées  

Detailed subject:  
Buildings renovation is a major issue in the international context of the reduction of consumption and of greenhouse gas emissions.  

In France, the office buildings energy retrofit challenge represents between 165 and 197 million m² for entire investments, reaching 190 billion euros. Spread out until 2022 it represents an annual expenditure of about 19 billion euros. This change would represent the creation of approximately 330,000 jobs (on a basis of an average of 60,000 €/an). This challenge would forecast important economic consequences, revenues from taxes and social contributions.  

However, so that the projects of thermal renovation technically have a measurable impact on energy consumptions, and indoor comfort, it appears necessary to make studies upstream (in design phase) allowing:  

- to identify and treat on a hierarchical basis the solutions to be implemented (opaque glazing, walls, ventilation…)  
- to study the interest of any materials and the impact of their implementation (thermal inertia, phase change materials…)  

To make these studies, the recourse to simulation is unavoidable. The use of traditional tools for direct simulation (TRNSYS, Energy +, Comfie) runs up here against the difficulty in working on an existing building.
The proposed work within the framework of the proposition is to develop a multicriteria tool for energy renovation of old buildings.

Methodology will be based on work already carried out (in numerical simulation) within the framework of project ANR-HABISOL (2009-2012) AMMIS (Multicriteria analyses and inverse method in energy simulation for buildings), in particular the use of algorithms based on the reflective Newton method.

The idea is here to propose to engineers and architects a tool that will allow establishing a hierarchy of several improvements to consider, according to energy, but also comfort criteria.

The study will focus primarily on improvements to the building envelope and its materials. Inverse method will be used to come across the characteristics of the building walls again. Based on a consumption target, improvements will be identified on various items (windows, insulation, and ventilation). A study will bring out a multi-criteria tool to assess the impact of these solutions.

**Candidate profile:**

- Thermal building modeling
- Numerical analysis
- Inverse problems
- Building technology notions

**Related publications**
Numerical and experimental identification of simplified building walls using an reflective-Newton inverse method

K. Limam a,*, T. Bouache b, S. Ginestet c, G. Lindner a

a Laboratory of engineering sciences for environment (LaSIE), University of La Rochelle, France

b Université de Bordeaux, Université Bordeaux1-Arts et Métiers Paristech-ENSCPB-CNRS, Laboratoire I2M UMR 5295, Talence, France

c Université de Toulouse, INSA-Université Paul Sabatier, LMDC EA 3027, Toulouse, France

Abstract

In this article the coupling of a direct thermal calculation with an optimization algorithm is presented, in order to achieve the identification of the thermal characteristics of a simplified building structure. The resolution of the direct thermal calculation is based on an electric circuit representation, solved using a numerical solution using the finite differences method. The optimization model minimizes a criterion such as « least squares » between the desired temperatures inside the building and the model response, time domain, by an inverse iterative algorithm « Reflective Newton ». The proposed optimization model is then validated with an experimental case, a closed wooden structure with one side being heated.

Keywords: Building, Thermal, Optimization, Experimental, Reflective Newton.

Introduction

With a strong presence in all developments of thermal regulation, the thermal design of buildings (including the walls and the envelopment) is important because it should control the amount of energy required to ensure thermal comfort throughout the year. The control of this parameter depends, in large part by an appropriate choice of materials (type, dimensions ...) which constitute the building’s wall.

Many works are dedicated to the optimization of thermal insulation and building walls. Mahlia et al [1] have established a correlation between the thermal conductivity of the insulation and optimal thickness in the form of a second order polynomial. Comakli and Yuksel [2] determined the optimum thickness of insulation for exterior walls based on the life cycle of buildings in the colder cities of Turkey. Al-Khawaja [3] determined for each type of insulation the optimum thickness, using as the optimization criterion the total cost of the energy consumed and isolation on hot countries. Al-Sanea et al [4] had determined with a
dynamic model of thermal transfer the effects of the electricity tariff over the optimal thickness of a building isolation in Saudi Arabia. Lollina et al. [5] conducted a study to determine the best level of insulation in new buildings from the energy, economic and environmental point of view.

<table>
<thead>
<tr>
<th>Nomenclature</th>
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<tbody>
<tr>
<td>C</td>
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<td>e</td>
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<tr>
<td>H</td>
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<td>Q</td>
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<td>R</td>
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<td>t</td>
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<tr>
<td>T</td>
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<td>(T_0)</td>
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<td>(T_1)</td>
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<td>(\Delta t)</td>
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<th>Greek symbols</th>
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<td>(\phi)</td>
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<table>
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<th>Subscripts and superscripts</th>
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<tr>
<td>E</td>
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</table>

The role of the thermal inertia of the building is a topic widely studied in the literature. Balaras [6] for example, has highlighted the role of thermal mass on the cooling load of a building. He also did in this study a large review and classification of simulation tools for calculating the thermal load and air temperature inside a building, and taking into account the effect of thermal inertia. Asan and Sancaktar [7] showed that the thermophysical properties of the wall have a significant effect on the delay and damping of the thermal wave. K. Ulgen [8] for his part initiated a theoretical and experimental study on the effect of thermophysical properties of the walls on the delay and damping of the building response. He suggested the use of multilayer walls with insulation for buildings occupied all day and monolayer walls for buildings occupied during specific time intervals.

To characterize the dynamics of the building, Antonopoulos and Koronaki [9] defined an apparent capacitance and effective capacitance. Other authors have demonstrated the
importance of the position of the insulating layer in the wall on the dynamic behavior of buildings (d’Asan [10] and [11], and Bojic and Loveday [12]). These authors have analyzed the influence of the insulation/masonry distribution in a wall with three layers over the energy consumption for heating and cooling.

McKinley et al [13] presented a procedure for optimizing the thermal parameters of a building (thermal resistance and thermal inertia). The direct model is solved numerically and the optimization is performed by the algorithm “Reflective-Newton”, available in a library of MatLab. Sambou et al [14] developed a model based on the thermal quadrupole method coupled with a genetic algorithm evolutionary multi-objective. The objective of their work was to find the best compromise between thermal insulation and thermal inertia of a wall. The solutions are presented as a Pareto front (a set of non-dominated solutions, solutions among which we cannot decide whether a solution is better than another, since there is no single systematically lower than the others on all objectives).

Our model is restricted to the study of the structure of a building with glass located in Gironde (France), subject to all the outer walls at a temperature and a flow of heat over a period of one year. A representation by an equivalent circuit diagram was proposed and solved numerically by the finite difference method. The interest of our study lies in the method of building thermal optimization (characterized by thermal, thermal capacity). The method involves minimizing a criterion of type "least squares", between the desired temperature inside the room and the response of the model.

**Direct model presentation**

We consider a structure (Fig. 1), consisting of six homogeneous walls, separating an external environment temperature $T_E(t)$ from an internal environment assumed isothermal (thermal capacity of the indoor air $C_{IN}$). It exchanges with the external environment by convection and linearized radiation (exchange coefficient $H_E = 1/R_E$) and absorbs a heat flow from the sun radiation ($\phi_s(t)$). So it exchanges with the indoor environment by convection and linearized radiation (heat transfer coefficient $H_{IN} = 1/R_{IN}$) and absorbs heat flux from a heating source ($\phi_C(t)$).
Fig 1. building outline and electrical analogy

The conduction exchanges treated in steady state (the case of glazing) are represented using only a single conductance. It is an analog model type 1R. The conductive exchange within the walls is considered in transient state. We use here the analog models 1R2C.

The heat equations on the nodes $T_{IN}$, $T_1$ and $T_0$ are written in the following form:

$$H_{IN}(T_{IN} - T_1) + H_T(T_{IN} - T_E) + C_{IN} \frac{dT_{IN}}{dt} = \phi_C(t) \quad (1)$$

$$C_M \frac{dT_1}{dt} + H_M(T_1 - T_0) + H_{IN}(T_1 - T_{IN}) = \phi_T(t) \quad (2)$$

$$C_E \frac{dT_0}{dt} + H_E(T_0 - T_E) + H_M(T_0 - T_1) = \phi_S(t) \quad (3)$$

Using the finite difference method for discretization, we obtain the following system of equations:

$$T_{n+1}^{IN} = a_1 T_n^{n+1} + b_1 T_1^n + c_1 T_{n+1}^E + d_1 \phi_C^n \quad (4)$$

$$T_1^n = a_2 T_1^{n+1} + b_2 T_0^n + c_2 T_{n+1}^{n+1} + d_2 \phi_T^n \quad (5)$$

$$T_0^n = a_3 T_0^{n+1} + b_3 T_1^n + c_3 T_{n+1}^{n+1} + d_3 \phi_S^n \quad (6)$$

$a_i$, $b_i$, $c_i$ et $d_{i=1...3}$ are coefficients which depend on geometric and thermophysical characteristics of the building (see Annex 1).

Inverse model selection

The thermal optimization of a building refers to the research of a better solution to optimize several variables of the thermal comfort under different constraints. The term « better » indicates that exist one or more solutions of conception. In a process of optimization, the variables are selected to describe the system (for example the size, the form, materials,
In general, a problem of optimization consists in minimizing one or more objective functions subject to certain constraints, it is written in this form:

$$\min_{\beta, D} [J_1(\beta), J_2(\beta), \ldots, J_m(\beta)]$$

(7)

Where $J_i (i = 1, \ldots, m)$ is an objective function, $\beta$ indicates the vector of parameter to be identified in the field of variable D.

Or in the case where we look for a single objective ($m = 1$), the function to be minimized (Eq. 7) can be written in the following form:

$$\min_{\beta} J(\beta), \quad l \leq \beta \leq u$$

(8)

The function (8) is optimized by the algorithm «Reflective Newton». This is an iterative algorithm applied to nonlinear functions with several variables subject to upper and lower bounds of the variables. Each iteration is to find an approximate solution of a large linear system using preconditioned conjugate gradients method. The details of this algorithm are given in the work of Coleman et al [15 and 16].

**optimization method**

In this paper the optimization method is to determine the set of thermophysical parameters of a building envelope, minimizing a quadratic criterion between the temperatures calculated by the direct model (Equations 4-6), and the temperatures recorded experimentally.

$$J(\beta) = \sum_{i=1}^{N} [T_{in}(\beta, t) - T_{mes}(t)]^2$$

(9)

The vector $\beta$ gathers the parameters to be estimated. The minimization of $J$ leading to the identification of the parameters is carried out with the algorithm “Reflective Newton”. The identification of parameters is carried out in two stages (Fig. 2). As a preliminary, one simulates random errors while adding the exact temperatures. The errors are represented by a Gaussian noise $\zeta$ with zero mean and unit variance, the standard deviation of noise is equal to (Equation 10).

$$T_{mes}(t) = T_{in}(t) + \zeta \sigma$$

(10)
Application of the optimization method

To solve the direct problem, we used the physical and thermophysical characteristics of the building given in Table 1. The thermal solicitations experienced by the wall in the room are generally periodic: the outside temperature, the solar radiation and the heating of the room follow a daily variation. They are generated by TRNSYS for the region of Gironde (Fig. 3 and 4).

<table>
<thead>
<tr>
<th>Simulation period</th>
<th>for 1 January to 31 December</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orientation</td>
<td>South</td>
</tr>
<tr>
<td>Fraction walls /</td>
<td>20% for all the building</td>
</tr>
<tr>
<td>windows</td>
<td>surfaces</td>
</tr>
<tr>
<td>Dimension</td>
<td>10 x 4 x 10 m³</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Thermal conductance (W/K)</th>
<th>( H_M )</th>
<th>( H_E )</th>
<th>( H_{IN} )</th>
<th>( H_T )</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>86.727</td>
<td>6560</td>
<td>1640</td>
<td>51.2</td>
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</table>

<table>
<thead>
<tr>
<th>Thermal capacity (J/K)</th>
<th>CM</th>
<th>CE</th>
<th>CIN</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>9.368 × 10⁶</td>
<td>8.183 × 10⁷</td>
<td>482880</td>
</tr>
</tbody>
</table>

Table 1. Physical characteristics of the building

Fig. 2. Solving algorithm

Fig. 3. solar flux variation for the considered building (1st January to 31st December)

Fig. 4. Outside temperature evolution considered building (1st January to 31st December)
Figures 5a and 5b show the result of the direct simulation of the dynamic evolution of the temperature inside the building for different noises (0, 0.1 and 0.5°C).

We seek to identify both the thermal conductance $H_i (W/K)$ and the thermal capacity $C_i (J/K)$ of the walls constituting the envelopment of the building. All the unknown parameters are grouped in the vector $\beta (H_M, H_T, C_M, C_E)$. The $H_i$ values are good between 1 and 1000 W/K, and the values of $C_i$ are between $10^4$ and $10^9$ J/K. The algorithm is initialized from $\beta^0 \{10, 10, 10^7, 10^7\}$.

The optimization algorithm runs the direct model to make converge the indoor temperature profile varying the unknown parameters in $\beta$, thanks to reflective Newton algorithm. The objective is to make meet the simulated indoor temperature of the direct case (with known layers composition) and the simulated indoor temperature from the simulations runs by the algorithm (with unknown layers composition, expressed in $\beta$).

For zero noise ($\sigma = 0$), the values identified ($H$ and $C$ in the Tables 2 and 3) are very close to exact values with a relative error close to zero. By contrast, both surface thermal capacity ($C_M$ and $C_E$) the relative error is approximately 24%. Indeed, the steady state temperature is not very sensitive to these two parameters, which define only the thermal inertia of the building (transient). Figure 7 clearly shows the sensitivity of the thermal response of these two parameters.

By adding first a slight noise to the vector indoor temperature, ($\sigma = 0.1$ and then $\sigma = 0.5$), we note that despite the oscillatory character of these temperatures, the values of thermal conductance are very close to the exact values with 3.10% error for the case more severe ($\sigma = 0.5$). The values identified of thermal capacity generate 25% error but are still acceptable in the construction field.

<table>
<thead>
<tr>
<th>$H_M (W/K)$</th>
<th>Relative error (%)</th>
<th>$H_T (W/K)$</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact value</td>
<td>86.727</td>
<td>51.200</td>
<td></td>
</tr>
<tr>
<td>Initial value</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$C_M (J/K)$</td>
<td>Relative error (%)</td>
<td>$C_E (J/K)$</td>
</tr>
<tr>
<td>-------------</td>
<td>---------------</td>
<td>-------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>0</td>
<td>$9.3685 \times 10^6$</td>
<td>13.11</td>
<td>$8.1482 \times 10^7$</td>
</tr>
<tr>
<td>0.1</td>
<td>$1.0597 \times 10^7$</td>
<td>15.51</td>
<td>$9.9843 \times 10^7$</td>
</tr>
<tr>
<td>0.5</td>
<td>$1.1214 \times 10^7$</td>
<td>19.70</td>
<td>$1.0007 \times 10^8$</td>
</tr>
</tbody>
</table>

Table 2. thermal conductance identification

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$C_M (J/K)$</th>
<th>Relative error (%)</th>
<th>$C_E (J/K)$</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$86.3211$</td>
<td>0.47</td>
<td>$51.325$</td>
<td>0.24</td>
</tr>
<tr>
<td>0.1</td>
<td>$87.083$</td>
<td>0.41</td>
<td>$50.9629$</td>
<td>2.37</td>
</tr>
<tr>
<td>0.5</td>
<td>$89.4191$</td>
<td>3.10</td>
<td>$49.952$</td>
<td>2.43</td>
</tr>
</tbody>
</table>

Table 3. thermal capacity identification

For a better understanding of the thermal conductance influence and the thermal capacity of the building structure on the internal temperature, we carried out a sensitivity study. The results are shown on Figure 7, representing the transitory evolution of reduced sensibility $H_M \left( \partial T_{IN} / \partial H_M \right)$, $H_T \left( \partial T_{IN} / \partial H_T \right)$, $C_M \left( \partial T_{IN} / \partial C_M \right)$ and $C_E \left( \partial T_{IN} / \partial C_E \right)$. We note that the internal temperature is not much sensible to $C_E$ and to a lesser degree to $H_M$ and $C_M$, what explains the difficulties encountered by the algorithm to identify the surface heat capacity $C_E$.

![Fig. 7. Temperature reduced sensibility for the parameters $H_M$, $H_T$, $C_M$ and $C_E$](image)

**Experimental case study**

To validate our approach of identification, we had developed an experimental cell, as well as an adapted instrumentation, allowing the recording of the rough temperatures of the walls, of the interior of the cell and finally the environment temperature. These measurements are required to feed the approach of identification by the inverse method.

The experimental cell possesses a cubic form of 59.6 cm edge. It consists of wooden panels (pine) without insulation, with the following properties: conductivity 0.14 W/mK, heat...
capacity 550 J/kg.K, and density equal to 2500 kg/m³. The panels used have a 1.8 cm thickness.

The southern surface of the cell is heated by two halogen lamps of a maximum power of 330W each one, placed at a specific angle and distance to achieve a homogeneous heating of 240 W/m² (Case 1) and 260 W/m² (Case 2). A fluxmeter was used in several positions to measure the exact heat flow of the surface and adjust the lamps position. The wall surface temperatures (interior and external face), are measured using the thermocouples of the type K with diameter 100 µm. The external temperature is measured east and west of the cell at a distance of 15 cm using a PT100 sensor. The internal temperature is measured by a PT100 also the center of the cell.

The signals of the thermocouples are recovered via a central data acquisition. The data acquisition is controlled by a micro-computer and is equipped with multiplexed cards. Our measures are taken every 60 seconds for 5 hours of test.

Fig. 8. General diagram of the experimental cell

We present on figures 9 and 10 the recording of the rough temperatures obtained for two heat flows (240 and 260 W/m²). These figures highlight a rise in temperature at interior of the cell, consequence of heat flow from the two lamps. For the first case we notice an increase of 4.5°C and the second case it rises to 5.5°C.

Fig. 9. Measured temperatures
(Ω_s = 240 W/m²)

Fig. 10. Measured temperatures
(Ω_s = 260 W/m²)
Experimental identification

The objective here is to identify the thermal properties of the experimental cell with homogeneous walls and surface heat flow from the lamps, by using the temperature measurements. The procedure of identification consists in searching an optimal set of parameters \( f_{opt}(H_M, C_M, Q_s) \), which minimizes the function objectifies \( \Delta(f) \), given by the equation 9. \( H_M, C_M \) are, respectively, thermal conductance and thermal capacity while \( Q_s \) is the imposed heat flux. \( \Delta(f) \) represents the standard deviation between the temperature calculated by the direct model (eq 4), and the temperature measured in the center of the experimental cell. The research of the optimal condition is based on the algorithm “Reflective Newton”. The results from the identification achieved from case 1 and case 2 are grouped in Table 5.

<table>
<thead>
<tr>
<th></th>
<th>( H_M (W/K) )</th>
<th>( C_M (J/K) )</th>
<th>( \Phi_S (W/m^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Initial</td>
<td>25</td>
<td>( 10^4 )</td>
<td>200</td>
</tr>
<tr>
<td>Identified</td>
<td>16.14</td>
<td>2.71 ( \cdot 10^4 )</td>
<td>243.89</td>
</tr>
<tr>
<td>Measured</td>
<td>15.6</td>
<td>2.48 ( \cdot 10^4 )</td>
<td>240</td>
</tr>
<tr>
<td>Relative error (%)</td>
<td>3.46</td>
<td>9.27</td>
<td>1.62</td>
</tr>
<tr>
<td>Case 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial</td>
<td>25</td>
<td>( 10^4 )</td>
<td>200</td>
</tr>
<tr>
<td>Identified</td>
<td>16.58</td>
<td>2.27 ( \cdot 10^4 )</td>
<td>243.98</td>
</tr>
<tr>
<td>Measured</td>
<td>15.6</td>
<td>2.48 ( \cdot 10^4 )</td>
<td>257</td>
</tr>
<tr>
<td>Relative error (%)</td>
<td>6.28</td>
<td>8.46</td>
<td>5.06</td>
</tr>
</tbody>
</table>

Table 5. Identification results

The results show a good agreement between the identified values and those measured, with a maximum relative variation which does not exceed 10%. We present on red figures 1 and 2, temperatures and the temperatures calculated by the direct model, starting from the initial and identified data file. It is noted that the temperatures calculated by the identified game are very close to the measured temperatures.

**Fig. 11. Temperature plot: Case 1**

**Fig. 12. Temperature plot: Case 2**

Conclusion

In this paper, a temperature calculation module for simple buildings is presented, based on the coupling of two models: direct model based on the electrical analogy and an inverse model based on the method of “reflective newton method” applied for nonlinear functions.
The module can estimate the thermal parameters of the walls and heating needs of the building. The interest of the module presented in this paper is to identify technical solutions that can meet the requirements. Direct simulation involved a large number of trials to achieve results (RT2012, [17]). The inverse simulation used in our module can quickly give a first guidance of the composition of the walls (design phase), which can then be taken up by direct simulations (TRNSYS, Energyplus, COMFIE, e.g.) to refine solutions. The module could be used by architects as an artifice of calculation, to understand and define in advance the better insulation in new construction. Further work will lead to initially identify more complex walls involving windows for instance. The longer-term objectives of the project are to optimize the design of the envelope to limit the consumption of heating while respecting the traditional criteria of thermal comfort.

The module was tested on an experimental bench. It made it possible to effectively identify the physical parameters of the envelope with controlled conditions and acceptable errors. The next stage is to propose configurations that matches with real buildings conditions, by adding ventilation (blowing, recovery, infiltration), and the contributions of heats (heating sources and occupation).

Acknowledgements

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References


Thermal identification of building multilayer walls using reflective Newton algorithm applied to quadrupole modelling

S. Ginestet\textsuperscript{a,}\textsuperscript{*}, T. Bouache\textsuperscript{b}, K. Limam\textsuperscript{c}, G. Lindner\textsuperscript{c}

\textsuperscript{a} Université de Toulouse, INSA, UPS, LMDC (Laboratoire Matériaux et Durabilité des Constructions), EA 3027, Toulouse, France
\textsuperscript{b} Université de Bordeaux, Université Bordeaux I-arts et Métiers ParisTech-ENSCPB-CNRS, Laboratoire I2M, UMR 5295, Talence, France
\textsuperscript{c} Laboratory of Engineering Sciences for Environment (LaSIE), University of La Rochelle, France

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\section{A B S T R A C T}

Designing low-energy buildings has become a necessity, encouraged by thermal regulations, the need for energy savings and environmental awareness. Computer-aided thermal design of building walls is currently investigated using the latest optimization algorithms. This paper studies building multilayer walls by coupling a direct thermal model with a specific optimization algorithm. The direct problem solution is based on the Laplace transform of the quadrupole method, and then translated by numerical inversion into the time domain by the Fourier series method. The optimization model minimizes a least squares criterion between intended indoor temperatures and a direct response model. The work aims to optimize the thermal insulation and the heat capacity of wall layers and further building heating loads. An indoor temperature evolution is specified under fixed outdoor conditions in order to identify the composition of the building walls using an inverse resolution based on a reflective Newton algorithm applied to a direct quadrupole model.

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\section{1. Introduction}

Computer-aided design for the thermal behaviour of buildings, including walls and insulation level, is now a requirement of thermal regulations in many places in the world. It leads to a forecast of the energy consumption to ensure thermal comfort over the whole year, sometimes even the whole life of the building. The thermal energy used depends strongly on the choices made for the walls of the buildings under study (e.g. their nature and size).

Many scientific works have already been carried out on the optimizing the thermal insulation of building walls. Recently, a correlation has been established between thermal conductivity of the thermal insulation and its optimal thickness through a second order polynomial [1]. Previously, the optimal thickness for external wall insulation had been determined by working on the building life cycle in the coldest cities of Turkey [2], and the best thickness for each kind of insulating material was identified from an economic point of view for very hot countries by considering both energy and insulating material costs [3]. Using dynamic thermal modelling, Al-Sanea et al. [4] have pointed out the effect of electricity prices on the optimal thickness of the insulating material in Saudia Arabia, and Lollini et al. [5] have run a new approach to determine the best insulation level for new buildings from the energy, economic and environmental standpoints. All these previous studies are based on the definition and optimization of the insulation. The thermal inertia, however, which appears crucial to the study of a building’s dynamic behaviour, is often neglected.

Nevertheless, the daily or seasonal impact of thermal inertia in buildings is developed in a few publications. For instance, Balaras et al. [6] underlined and improved knowledge of the impact of thermal inertia on cooling loads in a paper that also reviewed and classified several simulation tools for calculating cooling loads and predicting indoor air temperature by considering thermal inertia.

The major effect of the thermophysical properties of the wall materials on the magnitude and phase of a thermal wave applied to a building wall has been described and quantified [7], this approach being completed by a theoretical and experimental study on the effects of material thermal properties on the magnitude and phase of the whole building response [8]. The latter study suggests using multilayer insulated walls for buildings occupied throughout the year and monolayer walls for buildings occupied at specific times.

To describe a building’s dynamic behaviour, apparent and effective capacitances have been introduced [9]. Other authors have investigated the impact of the position of insulating material in the multilayer wall on the building’s dynamic behaviour [10–12]. In [12], the influence of the relative positions of insulation and masonry on heating and refrigeration consumption is quantified for a three-layer wall.
Thomas et al. [13] have used an original procedure to optimize the thermal resistance and thermal inertia of a typical building. The direct model is solved numerically and the optimization is achieved using an “interior – reflective Newton” algorithm. A reflective Newton method for solving non-linear minimization problems where some of the variables have upper and/or lower bounds was proposed by Coleman et al. [14,15], who established strong convergence properties. In particular, reflective Newton methods can achieve global and quadratic convergences. Experimental results for a quadratic objective function are provided in [14]. These computational results are extremely encouraging and indicate that reflective Newton methods are well suited to large-scale computations. A remarkable feature of this type of algorithm, illustrated by a typical example, is the very slow growth in the number of iterations required. Given a problem class and a “natural” way of increasing the problem dimension, reflective Newton methods appear to be strikingly insensitive to problem size [14].

Recently, a model has been developed based on thermal quadrupoles coupled to an evolutionary multi-objective genetic algorithm [16]. This work aims to find the best trade-off between the thermal insulation and thermal inertia of a building wall. Walls making the best trade-off between the two conflicting objectives are presented in a Pareto frontier. Optimal wall composition shows that the best disposition of layers is a massive layer on the indoor side and an insulating layer on the outdoor side. A new result obtained in this study is that the optimal thickness of the indoor massive layer is \( \Lambda/4 \) where the thermal wavelength \( \Lambda \) is an intrinsic parameter of the layer material depending on the period of oscillations. Our study was limited to a simplified building model providing periodic temperature and heat flux on all exterior walls.

In this study, a thermal quadrupole model is developed. As shown in [14], which also used the quadrupole method applied to simple buildings, in this paper, we take the thermal capacitance as a way to quantify the wall inertia. The temperature response of the model is transposed into a temporal field by Fourier numerical inversion. The interest of our study lies in the method for the thermal optimization of the multilayer building walls, characterized here by a thermal resistance, a thermal capacitance and a thickness. The method minimizes a least square criterion between expected indoor temperatures and model response. Based on a direct quadrupole model, an expected indoor temperature evolution is specified under outdoor fixed conditions. Assuming these conditions, the study aims to identify a building wall composition by using inverse resolution based on a reflective-Newton algorithm.

In the long term, the method is intended to be used by building designers to determine whether, energetically speaking, the initial sketch can be suitable, considering the physical materials available.

2. Assessment of a thermal building model based on thermal quadrupole modelling

The thermal zone considered (e.g. room of a building), made up of six homogenous walls, separates an outdoor environment (temperature \( T_{\text{ext}}(t) \), evolution given by [17]) from an isothermal indoor environment (thermal capacity of the media \( C_p \)). Walls exchange with the outdoor environment by convection (thermal outdoor resistance \( R_C \)) and absorb radiative thermal flux \( (\Phi_{\text{hv}}(t)) \) coming from the sun. On the other side (Fig. 1), walls exchange with the indoor environment by convection (thermal indoor resistance \( R_{\text{in}} \)) and absorb a thermal flux coming from heating systems \( (\Phi_{\text{HE}}(t)) \). Therefore, it is possible to calculate the time evolution of the indoor air temperature \( T_{\text{a}}(t) \), according to (6).

The objective of this paragraph is to detail the model used in this study. The data are physical and heating parameters of a building. The time-dependent inputs are outdoor conditions and indoor heating flux. The output is indoor air temperature here. This output is then compared to benchmark software results (e.g. TRNSYS) to validate the thermal zone model.

The heating conduction law and heating flux inside the materials are given by Eqs. (1) and (2), respectively.

\[
\rho C_p \frac{\partial T(x, t)}{\partial t} = \lambda \frac{\partial^2 T(x, t)}{\partial x^2} \tag{1}
\]

\[
\phi = -\lambda \frac{\partial T(x, t)}{\partial x} \tag{2}
\]

A Laplace transform applied to Eqs. (1) and (2), leads naturally to a relation between temperatures and heat flux through a transfer matrix, also well-known as the “wall thermal quadrupole” [18].

\[
\begin{bmatrix}
\hat{T}(p) \\
\hat{\phi}(p)
\end{bmatrix} =
\begin{bmatrix}
A(p) & B(p) \\
C(p) & A(p)
\end{bmatrix}
\begin{bmatrix}
\hat{T}_A(p) \\
\hat{\phi}_A(p)
\end{bmatrix} = M(p)
\begin{bmatrix}
\hat{T}_A \\
\hat{\phi}_A
\end{bmatrix} \tag{3}
\]

To introduce \( T_{\text{in}}, \Phi_D \) and \( \Phi_{\text{CH}} \), the available inputs, it can also be proved that (Fig. 1):

\[
\hat{T}_A = \frac{1}{\lambda A p} (\Phi_{\text{CH}} + \hat{\phi}_A) \tag{4}
\]

\[
\tilde{T}_{\text{in}} = \hat{T}_A + R_C(\hat{\phi}_R - \hat{\phi}_D) \tag{5}
\]

\[
\tilde{\phi}_E = \hat{\phi}_E - \hat{\phi}_A \tag{6}
\]

In formulas (3)–(6), \( \hat{T}(p) \) is the Laplace transform of temperature and \( \hat{\phi}(p) \) is the heat flux.
The subscripts \( E \) and \( A \) indicate respectively the outdoor and the indoor air on the two sides of the wall.

The final expression for the indoor temperature in the Laplace domain can be written as follows:

\[
\tilde{T}_I(p) = G(p)\tilde{T}_E(p) + H_{RA}(p)\tilde{\phi}_{CH}(p)
\]

(7)

with the following quantities:

\[
G(p) = \frac{1}{[A + BC_A p + R_E C + R_E D C_A p]}
\]

(8)

\[
H_{CH}(p) = R_E (C + DC_A p) G(p)
\]

(9)

\[
H_{RA}(p) = R_E G(p)
\]

(10)

In the general case, the wall is composed of \( N \) layers of material assumed to be in perfect thermal contact and the global thermal transfer matrix \( M(p) \) is the product of the thermal matrices associated with the several layers, including thermal transfer on the indoor wall face (Eq. (10))

\[
M(p) = M_0(p) \ldots M_k(p) \ldots M_I(p) M_m(p)
\]

\[
\begin{bmatrix}
1 & 1/h_m S_m \\
0 & 1
\end{bmatrix}
\]

(11)

with

\[
M_k(p) = \begin{bmatrix}
A_k(p) & B_k(p) \\
C_k(p) & A_k(p)
\end{bmatrix}
\]

(12)

\( M_k(p) \) is the quadrupole associated with the \( k \)th wall layer. The elements of this matrix are given in Eqs. (12)–(14).

\[
A_k(p) = D_k(p) = ch(\sqrt{pR_k C_k})
\]

(13)

\[
B_k(p) = R_k \cdot \frac{\sqrt{pR_k C_k}}{\sqrt{R_k C_k}}
\]

(14)

\[
C_k(p) = \frac{\sqrt{pR_k C_k}}{R_k} \cdot \frac{\sqrt{pR_k C_k}}{R_k}
\]

(15)

where \( p \), \( C_k = \rho_k C_{pl} \), \( e_k \) and \( R_k = e_k/\lambda_k \) are respectively the Laplace variable, the thermal capacity and the thermal resistance of the corresponding layer \( k \).

3. Model validation using TRNSYS

The simplified building model proposed here uses the concept of a “single area” with uniform temperature. In order to compare and then validate the model with TRNSYS [19], we assumed premises with a volume similar to our case. TRNSYS has been a world reference in building simulation tools for decades and has been benchmarked many times. All walls were composed of a brick layer, 30 cm-thick, and an outside insulation layer. The room exterior walls were subjected to a constant temperature \( T_{\infty} = 20^\circ \text{C} \) and flux \( \phi_{rad} = 1000 \text{ W/m}^2 \). Convective and radiative transfers were combined in the overall transfer coefficients with \( h_{out} = 20 \text{ W/m}^2 \) for outdoor air and \( h_{in} = 5 \text{ W/m}^2 \) K for air inside the room.

The comparison between the simplified model and TRNSYS was performed for two thicknesses of insulation: 2 and then 5 cm. The results of this comparison are shown in Fig. 2. We observe that the results of both models converge in terms of time constant and temperature values, for several insulation thicknesses. The observed differences are negligible, and so they are not representative in a building case study.

4. Selected optimization method

Thermal optimization of a building generally consists of searching for an optimal solution in terms of thermal comfort, for a set of variables satisfying a few constraints. The word “optimal”
suggests that several designs are suitable. In an optimization process, variables are selected to describe the system (e.g., size, shape, materials). The objective is to minimize or maximize a function (indoor air temperature difference in our study) and the constraints are linked to a working domain, which indicates a restriction or a limitation on a technological capacity of the system.

Generally, an optimization problem consists of minimizing one or more “objective functions”, with imposed constraints. It can be written as follows:

$$\text{minimize}_{x \in D} [f_1(x), f_2(x), \ldots, f_m(x)]$$

where $f_i \ (i = 1, \ldots, m)$ is an objective function and $x$ is the parameter vector to be identified in the domain $D$. In the case where only one objective is involved ($m = 1$), the function to be minimized (Eq. (16)) becomes:

$$\text{minimize}_{x, l \leq x \leq u} f(x) \quad (17)$$

Eq. (17) is solved using the reflective Newton algorithm. This is an iterative algorithm applied to non-linear multivariable functions, limited to conditions of the upper and lower boundary variables. Each iteration aims to find a quasi solution of a higher linear system using a preconditioned conjugate gradients method. More details can be found in [14,15], where the authors propose a reflective Newton method for solving non-linear minimization problems where some of the variables have upper and/or lower bounds.

In this paper, the optimization method aims to determine the set of unknown building physical parameters by minimizing a quadratic criterion between temperatures estimated by the quadrupole model and the desired temperatures (“experimental data”). The main way to obtain these expected indoor temperature variations, here, is to compute a theoretical evolution, assuming perfect control of this temperature on site, since, for new buildings that have not been built yet; we cannot measure the temporal evolution of indoor temperature directly.

$$J(\beta) = \sum_{t=1}^{N} [T_{\text{int}}(\beta, t) - T_{\text{mes}}(t)]^2$$

(18)

$\beta$ is the vector computing all the parameters to be estimated. The minimization of $J(\beta)$ leads to an identification of the parameters thanks to the reflective Newton algorithm. Identification of the parameters is achieved in two steps (Fig. 3). Firstly, random errors are simulated by adding Gaussian noise, $\xi$, of zero mean and unitary variance, to exact temperatures. The deviation of the noise is $\sigma$ (Eq. (19)). The second step uses the identification algorithm to minimize the quadratic function (Eq. (18)).

$$T_{\text{mes}}(t) = T_{\text{in}}(t) + \xi \sigma.$$  

(19)

5. Sensitivity analysis and results

The wall considered has a fixed wall thickness and consists of three layers of materials (Fig. 4): insulation (interior), brick and plaster (interior). The physical properties of three materials are summarized in Table 1.

The thermal stresses ($T_{\infty}$, $\phi_{RA}$, and $\phi_{CH}$) on the wall often have a periodic profile: the outdoor dry temperature, solar radiation and heating flux follow a quasi periodic daily variation [18]. The choice of periodic thermal loads is justified here by the resulting ease with which the equations can be written in the Laplace domain. However, in practice, if real meteorological data are taken, problems appear in processing the Laplace domain to the time domain.

$$T_{\infty} = \frac{\gamma_0}{p} + \frac{\gamma_1 \omega_1}{p^2 + \omega_1^2} + \frac{\gamma_2 \omega_2}{p^2 + \omega_2^2}$$

(20)

within the case studied:

$$\gamma_0 = 7 \text{ } ^\circ \text{C}, \quad \gamma_1 = 10 \text{ } ^\circ \text{C}, \quad \gamma_2 = 4 \text{ } ^\circ \text{C}, \quad \omega_1 = 2\pi/3600/24/362 \text{ rad s}^{-1} \quad \text{and} \quad \omega_2 = 365\omega_1$$

$$\tilde{\phi}_{RA} = \phi_{RA} \exp(-\text{HLS}\rho) \sum_{n=1}^{M} \left[e^{(-p\omega_2)}\frac{1}{p^2 + (\pi/\omega_1)^2}\right]$$

(21)

Fig. 4. Heating evolutions.

Fig. 5. Comparison with TRNSYS.

Fig. 6. Solving algorithm.

Fig. 7. Multilayer wall, outside insulation.
Table 1
Physical properties.

<table>
<thead>
<tr>
<th></th>
<th>e (cm)</th>
<th>λ (W/m·K)</th>
<th>c_p (J/kg·K)</th>
<th>ρ (kg/m³)</th>
<th>R (m²·K/W)</th>
<th>C (kJ/m³·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick</td>
<td>20</td>
<td>1.5</td>
<td>800</td>
<td>2500</td>
<td>0.134</td>
<td>400</td>
</tr>
<tr>
<td>Plaster</td>
<td>4</td>
<td>0.727</td>
<td>820</td>
<td>1600</td>
<td>0.055</td>
<td>52.48</td>
</tr>
<tr>
<td>Insulation</td>
<td>5</td>
<td>0.043</td>
<td>840</td>
<td>91</td>
<td>1.163</td>
<td>3.822</td>
</tr>
</tbody>
</table>

![Fig. 8. Simulated indoor temperatures.](image)

![Fig. 9. Thermal resistances identification.](image)

\[
\tilde{\phi}_{CH} = \phi_{CH}^{\text{max}} e^{(-\lambda_{AC} \rho)} \sum_{m=1}^{M} \left[(-1)^{m+1} e^{-\lambda_{AC} \rho m}\right] \times \left[\frac{1}{p} - \frac{1}{p + (K/\tau_1)}\right] \tag{22}
\]

To transport from the Laplace domain to the temporal domain, we use the Fourier Inversion. Figs. 5–7 represent the evolution of outdoor temperature, solar radiation and heating flux after the inversion in the temporal domain.

Before using the identification algorithm, we use the analytical solution (Eqs. (7)–(10)) to simulate temperatures, adding Gaussian noise (σ = 0, σ = 0.1 and 0.5). These temperatures are used to minimize the least square criterion (Eq. (18)). The simulation results are given in Fig. 8.

![Fig. 10. Sensitivity to thermal resistance.](image)

**Case 1 (\(\sigma\)).** Identification of thermal resistances \(R\) [m²·K/W].

This first study simply estimates the thermal resistances of the several layers making up the wall under consideration. All resistances, unknown parameters, are added to the vector \(\beta(R_{\text{brick}}, R_{\text{plaster}}, R_{\text{insulation}})\). The objective is to identify the \(R_i\) values within the 0.01–2.2 m²·K/W interval. These values are fixed by French thermal regulation 2005 (TR2005, [19]). The iterative process is initialized at the vector value \(\beta^0(1,1,1)\) [m²·K/W].

The results of the identification process are shown in Fig. 9. We observe that, despite the oscillatory character of the measured temperatures (effect of the noise addition), with the exception of the plaster thermal resistance, the values of thermal resistances are identified near the exact values. However identification errors are greater for higher noise, which was to be expected. There is a maximum relative error of 2.5% for the thermal resistance of the brick for high noise (σ = 0.5) and 1% for lower noise (σ = 0.1).

To better understand the influence of the thermal resistances of the layers on the temperature inside the building, we conducted a study of temperature sensitivity versus thermal resistance of the three layers. The results are shown in Fig. 10; the transient evolution of reduced sensitivity \(R_i(\partial T_m(t)/\partial R_i)\) is shown. Note that the indoor temperature is very sensitive to the thermal resistance of the insulation, followed by the brick, and finally the plaster, which explains the difficulties encountered by the algorithm in identify-
The tendencies identified in Fig. 14 indicate that the thermal resistance of plaster. This trend confirms physical tendencies found previously. For instance, in the case of thermal resistance identification, the sensitivity of the results is very high for the material that has the highest thermal resistance. The physical trends agree with the preliminary results, which was to be expected for a primary simulation.

**Case 2. Identification of thermal capacitances C [kJ/m² K].**

The second step of this study is to estimate the thermal capacitances of the several layers composing the wall considered: $\beta (C_{\text{brick}}, C_{\text{plaster}}, C_{\text{insulation}})$ with $C_i = R_i \cdot \varepsilon_i$. The objective is to identify the $C_i$ values within the 1–500 kJ/m² K interval. The iterative process is initialized at the vector value $\beta^0 (10, 10, 10)$ [kJ/m² K].

The results are presented in Fig. 11 for three increasing noise levels ($\sigma = 0, \sigma = 0.1$ and $\sigma = 0.5$). Apart from the $C_{\text{insulation}}$ values in the case of high noise, all the other values are very close to the identified target values.

When high noise is assumed in identifying the parameters, a maximum error of 18% is present on the $C_{\text{insulation}}$ value. Fig. 12 shows reduced sensitivities $C_i (\partial T_i (t)/\partial C_i)$ versus $C_i$ during the simulation time. These results confirm that the temperature inside the building is more sensitive to the brick capacitance and much less sensitive to the insulation. In other words, the building thermal inertia is conditioned by the brick layer heat capacity.

**Case 3. Identification of materials (R and C).**

The last step of this study is the estimation of both the thermal resistances and the thermal capacitances of the several layers constituting the wall. All the unknown parameters can be aggregated in the vector $\beta (R_{\text{brick}}, R_{\text{plaster}}, R_{\text{insulation}}, C_{\text{brick}}, C_{\text{plaster}}, C_{\text{insulation}})$. Once again, the $R_i$ values are within the 0.01–2.2 m² K/W interval and the $C_i$ values are within the 1–500 kJ/m² K interval. The final objective is to identify all the $R_i$ and $C_i$ values. The iterative process is initialized at the vector value $\beta^0 (1, 1, 1, 10, 10, 10)$.

For zero noise ($\sigma = 0$), $R_i$ and $C_i$ values are both identified (Figs. 13 and 14) and are very close to the target values, with a relative error close to zero. However, for a stronger noise ($\sigma = 0.1$ and $\sigma = 0.5$), the identification is more difficult and even impossible as it includes the thermal resistance of the plaster and the capacitance of the insulation. On the one hand, the thermal resistance of plaster (0.055 m² K/W) is weak compared with those of brick (0.134 m² K/W) and insulation (1.163 m² K/W). On the other hand, the thermal superficial capacitance of the insulation, 3.822 kJ/m² K, is lower than those of brick (400 kJ/m² K) and plaster (52.48 kJ/m² K), making these parameters impossible to estimate.

**Case 4. Identification of heating.**

In our study, the heating flux is assumed to be a periodic function (theoretical approach). The heating evolution is then governed by Eq. (18).

To identify the evolution of the heating flux, it is necessary to identify the parameters $\tau_1$ and $\phi_{\text{heat}}^{\max}$, which are the heating period and the daily maximum heating flux power respectively.

For instance, let us assume we wish to identify both the thermal resistances $R_i$ of the several layers and the heating flux driven by $\tau_1$ and $\phi_{\text{heat}}^{\max}$.

All the unknown parameters are grouped together in the vector $\beta (R_{\text{brick}}, R_{\text{plaster}}, R_{\text{insulation}}, \tau_1, \phi_{\text{heat}}^{\max})$. $R_i$ values are kept between 0.01 and 2.2 m² K/W, $\phi_{\text{heat}}^{\max}$ values are from 7 to 14 kW. $\tau_1$ values are from 8 to 14 h. The algorithm is initialized to the value $\beta^0 (1, 1, 1, 10, 8)$.

The results of the identification process are shown in Figs. 15 and 16. The values of thermal resistances are identified near
6. Conclusion

In this paper, a temperature calculation model for simple buildings is presented, based on the coupling of two models: a direct model using the thermal quadrupole method and an inverse model based on the reflective Newton method applied for non-linear functions. The model can estimate the thermal parameters of the walls and the building heating parameters, as defined by the model used. The interest of the model presented in this paper is that it identifies technical solutions that can meet the present requirements of society. Direct simulation involves a large number of trials to achieve the results (TR2005, [20]). The inverse simulation used in our model can quickly give a first guide to the wall composition, which can then be taken up by direct simulations (e.g. TRNSYS, Energyplus, COMFIE,) to refine the solutions. The model will be used as a calculation device by architects, to define the composition of the walls of a renovated construction or propose new insulation configurations for new projects. Further work will initially identify more complex walls, involving windows for instance. The long-term objectives of the project are to optimize the envelope design to limit heating consumption while respecting the traditional criterion of thermal comfort.

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References